General Certificate of Education
Advanced Level Examination
June 2012

Mathematics MS2B

Unit Statistics 2B

Thursday 21 June 2012 1.30 pm to 3.00 pm

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

1 At the start of the 2012 season, the ages of the members of the Warwickshire Acorns Cricket Club could be modelled by a normal random variable, X years, with mean μ and standard deviation σ.

The ages, x years, of a random sample of 15 such members are summarised below.

$$\sum x = 546 \quad \text{and} \quad \sum (x - \bar{x})^2 = 1407.6$$

(a) Construct a 98% confidence interval for μ, giving the limits to one decimal place. *(6 marks)*

(b) At the start of the 2005 season, the mean age of the members was 40.0 years.

Use your confidence interval constructed in part **(a)** to indicate, with a reason, whether the mean age had changed. *(2 marks)*

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2 The times taken to complete a round of golf at Slowpace Golf Club may be modelled by a random variable with mean μ hours and standard deviation 1.1 hours.

Julian claims that, on average, the time taken to complete a round of golf at Slowpace Golf Club is greater than 4 hours.

The times of 40 randomly selected completed rounds of golf at Slowpace Golf Club result in a mean of 4.2 hours.

(a) Investigate Julian's claim at the 5% level of significance. *(6 marks)*

(b) If the actual mean time taken to complete a round of golf at Slowpace Golf Club is 4.5 hours, determine whether a Type I error, a Type II error or neither was made in the test conducted in part **(a)**. Give a reason for your answer. *(2 marks)*

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3 The continuous random variable X has a cumulative distribution function defined by

$$F(x) = \begin{cases} 0 & x < -5 \\ \dfrac{x+5}{20} & -5 \leqslant x \leqslant 15 \\ 1 & x > 15 \end{cases}$$

(a) Show that, for $-5 \leqslant x \leqslant 15$, the probability density function, $f(x)$, of X is given by $f(x) = \frac{1}{20}$. *(1 mark)*

(b) Find:

(i) $\mathrm{P}(X \geqslant 7)$; *(1 mark)*

(ii) $\mathrm{P}(X \neq 7)$; *(1 mark)*

(iii) $\mathrm{E}(X)$; *(1 mark)*

(iv) $\mathrm{E}(3X^2)$. *(3 marks)*

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4 A house has a total of five bedrooms, at least one of which is always rented.

The probability distribution for R, the number of bedrooms that are rented at any given time, is given by

$$\mathrm{P}(R = r) = \begin{cases} 0.5 & r = 1 \\ 0.4(0.6)^{r-1} & r = 2,\ 3,\ 4 \\ 0.0296 & r = 5 \end{cases}$$

(a) Complete the table below. *(2 marks)*

(b) Find the probability that fewer than 3 bedrooms are **not** rented at any given time. *(3 marks)*

(c) (i) Find the value of $\mathrm{E}(R)$. *(2 marks)*

(ii) Show that $\mathrm{E}(R^2) = 4.8784$ and hence find the value of $\mathrm{Var}(R)$. *(3 marks)*

(d) Bedrooms are rented on a monthly basis.

The monthly income, £M, from renting bedrooms in the house may be modelled by

$$M = 1250R - 282$$

Find the mean and the standard deviation of M. *(3 marks)*

r	1	2	3	4	5
$\mathrm{P}(R = r)$	0.5				0.0296

5 (a) The number of **minor** accidents occurring each year at RapidNut engineering company may be modelled by the random variable X having a Poisson distribution with mean 8.5 .

Determine the probability that, in any particular year, there are:

(i) at least 9 minor accidents; *(2 marks)*

(ii) more than 5 but fewer than 10 minor accidents. *(3 marks)*

(b) The number of **major** accidents occurring each year at RapidNut engineering company may be modelled by the random variable Y having a Poisson distribution with mean 1.5 .

Calculate the probability that, in any particular year, there are fewer than 2 major accidents. *(2 marks)*

(c) The **total** number of minor and major accidents occurring each year at RapidNut engineering company may be modelled by the random variable T having the probability distribution

$$P(T = t) = \begin{cases} \dfrac{e^{-\lambda}\lambda^t}{t!} & t = 0, 1, 2, 3, \ldots \\ 0 & \text{otherwise} \end{cases}$$

Assuming that the number of minor accidents is independent of the number of major accidents:

(i) state the value of λ ; *(1 mark)*

(ii) determine $P(T > 16)$; *(2 marks)*

(iii) calculate the probability that there will be a total of more than 16 accidents in each of at least two out of three years, giving your answer to four decimal places. *(3 marks)*

6 Fiona, a lecturer in a school of engineering, believes that there is an association between the class of degree obtained by her students and the grades that they had achieved in A-level Mathematics.

In order to investigate her belief, she collected the relevant data on the performances of a random sample of 200 recent graduates who had achieved grades A or B in A-level Mathematics. These data are tabulated below.

		Class of degree				
		1	**2(i)**	**2(ii)**	**3**	**Total**
A-level grade	**A**	20	36	22	2	80
	B	9	55	48	8	120
	Total	29	91	70	10	200

(a) Conduct a χ^2 test, at the 1% level of significance, to determine whether Fiona's belief is justified. *(9 marks)*

(b) Make **two** comments on the degree performance of those students in this sample who achieved a grade B in A-level Mathematics. *(2 marks)*

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7 A continuous random variable X has probability density function defined by

$$f(x) = \begin{cases} \frac{1}{6}(4 - x) & 1 \leqslant x \leqslant 3 \\ \frac{1}{6} & 3 \leqslant x \leqslant 5 \\ 0 & \text{otherwise} \end{cases}$$

(a) Draw the graph of f on the grid on page 6. *(2 marks)*

(b) Prove that the mean of X is $2\frac{5}{9}$. *(4 marks)*

(c) Calculate the **exact** value of:

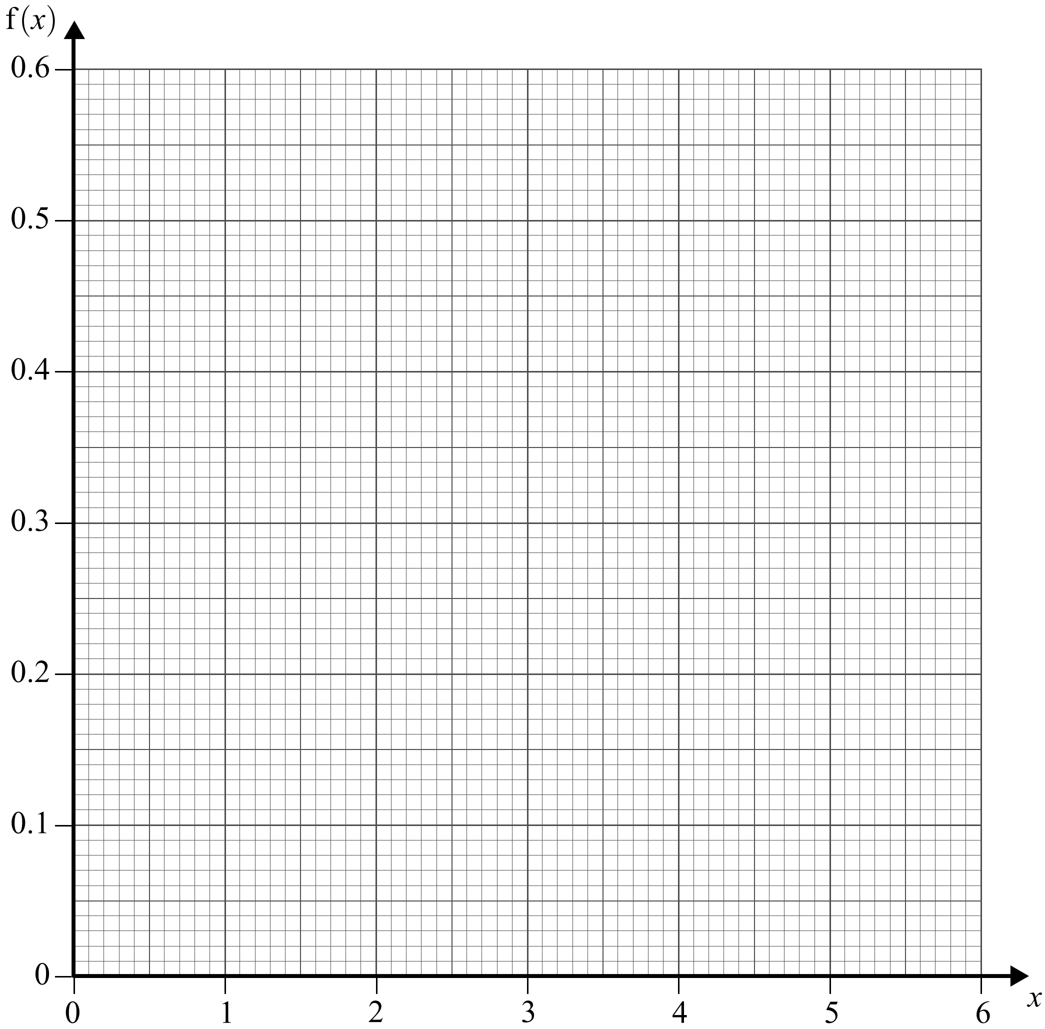
(i) $P(X > 2.5)$; *(2 marks)*

(ii) $P(1.5 < X < 4.5)$; *(3 marks)*

(iii) $P(X > 2.5$ **and** $1.5 < X < 4.5)$; *(2 marks)*

(iv) $P(X > 2.5 \mid 1.5 < X < 4.5)$. *(2 marks)*

Turn over ▶


f(x)
0.6
0.5
0.4
0.3
0.2
0.1
0
0
1
2
3
4
5
6
x